# Comparative Analysis between Different Approaches for Electrical Properties Tomography

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**This paper presents a comparative analysis between three numerical methods used in the electrical properties tomography to reconstruct the conductivity and permittivity of biological tissues from the radiofrequency magnetic field map measurable in a magnetic resonance system. The analysis is performed on model problems, whose solution provides the magnetic field values to be used as virtual measurements. The effects on the result accuracy of very high and very low conductivity materials (mimicking the presence of medical implants) are finally evaluated.**

*Index Terms*—**Electric properties tomography, Magnetic resonance imaging, Maxwell equations, Numerical simulation**

## I. INTRODUCTION

ELECTRICAL properties tomography (EPT) reconstructs the electric conductivity and permittivity of biological tissues from radiofrequency magnetic field distributions measurable in a magnetic resonance (MR) scanner. Unlike other reconstruction techniques, as the MR electrical impedance tomography (MREIT) [1]-[2], EPT avoids the use of additional instrumentations over the MR scanner. Starting from Maxwell equations, different numerical approaches with different simplifying assumptions have been proposed to determine a map of the electrical properties inside a human body [3]-[5]. This paper presents a comparative analysis between three reconstructive EPT methods, to put in evidence advantages and drawbacks. As a first approach, we solve a model problem, where a linearly polarized plane wave normally incidences to the surface of stratified structure. The magnetic field values given by the solution of the electromagnetic problem [6] are assumed as virtual measurements for the reconstructive methods. Finally, the paper evaluates the result accuracy in presence of materials with very high (metals) or very low (ceramic) conductivity, mimicking the presence of a prosthesis.

## II. RECONSTRUCTIVE METHODS

All the considered methods for EPT come from the differential equation for the magnetic field $\boldsymbol{H}$, written in frequency domain by exploiting the phasor notation

$$-\nabla^2 \boldsymbol{H} = \frac{1}{\tilde{\varepsilon}} \nabla \tilde{\varepsilon} \times (\nabla \times \boldsymbol{H}) + \omega^2 \mu_0 \tilde{\varepsilon} \, \boldsymbol{H} \,, \qquad (1)$$

where $\omega$ is the angular frequency, $\mu_0$ is the vacuum magnetic permeability and $\tilde{\varepsilon} = \varepsilon_0 \varepsilon_{\mathrm{r}} - \mathrm{j}\, \sigma/\omega$ is the complex permittivity (the EPT problem unknown), being $\varepsilon_0$ the vacuum electric permittivity, $\varepsilon_{\mathrm{r}}$ the relative permittivity and $\sigma$ the equivalent conductivity of the medium.

Method 1 disregards the first addendum of the right hand side of (1) and refers to the classical Helmholtz equation for the magnetic field in homogeneous media. Denoting with $H$ a measurable component of the magnetic field, Method 1 leads to the simple relation $\tilde{\varepsilon} = -\nabla^2 H/(\omega^2 \mu_0 H)$. The same relation arises approximating the integral relation in [3] with the first order Newton-Cotes quadrature formula.

Method 2 considers the complex permittivity and its spatial derivatives in (1) as algebraic independent unknowns [5]. The measurements of the magnetic fields induced by linearly independent sources allow us to get the distribution of the complex permittivity by solving, in every point of interest, a small linear system.

Finally, Method 3 elaborates (1) as a partial differential equation with respect to the unknown $\tilde{\varepsilon}$ [4]. Multiplied by $u = \tilde{\varepsilon}^{-1}$, (1) becomes the linear convection-reaction equation

$$\nabla u \times (\nabla \times \boldsymbol{H}) - u \, \nabla^2 \boldsymbol{H} = \omega^2 \mu_0 \boldsymbol{H} \,. \qquad (2)$$

Both Methods 2 and 3 use (1) without further approximations, so requiring the knowledge of the curl of the magnetic field, computable only when knowing all the magnetic field components.

## III. ONE DIMENSIONAL MODEL PROBLEM

In order to test the accuracy of the reconstructive methods in presence of wide spreads in electrical properties (as the ones due to prosthesis), some simulations are performed on a one-dimensional problem, where a linearly polarized plane wave impinges normally to the surface of a stratified structure. Given the electrical properties of each material in the structure, the magnetic field can be analytically computed [6]. The magnetic field values on some points are used as virtual measurements and applied in the EPT methods. The reconstructed electrical properties are then compared with the actual ones. All the proposed methods need first and/or second derivatives of the measured magnetic fields, that are approximated with a finite difference approach starting from the virtual measurements. This requirement stresses the need of a finer and finer grid for the magnetic field measurements.

The unknowns for Method 2 in the 1-D problems are the complex permittivity and its spatial derivative. Thus, only two linearly independent magnetic fields are necessary (i.e. the two propagating in opposite directions). Methods 1 and 2 are applied in all the points of the virtual measurements; no further interpolations are performed. In Method 3, this first 1-D model permits to express (2) in the conservative form

TABLE I
STRUCTURE A

| Actual values | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| ELECTRIC CONDUCTIVITY (S/m) | | | |
| 0.0369 | 0.0369 | 0.0369 | 0.0368 |
| 0.7192 | 0.7192 | 0.7195 | 0.7189 |
| 0.0674 | 0.0673 | 0.0674 | 0.0673 |
| RELATIVE ELECTRIC PERMITTIVITY | | | |
| 5.922 | 5.922 | 5.922 | 5.922 |
| 63.47 | 63.47 | 63.45 | 63.48 |
| 14.72 | 14.72 | 14.72 | 14.72 |

TABLE II
STRUCTURE B

| Actual values | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| ELECTRIC CONDUCTIVITY (S/m) | | | |
| 0.0369 | 0.0369 | 0.0369 | 0.0368 |
| 0.7192 | 0.7192 | 0.7195 | 0.7189 |
| $2.6 \cdot 10^{-6}$ | 6.591 | 0.0716 | 0.0461 |
| 0.0674 | 0.0673 | 0.0674 | 0.0673 |
| RELATIVE ELECTRIC PERMITTIVITY | | | |
| 5.922 | 5.922 | 5.922 | 5.922 |
| 63.47 | 63.47 | 63.45 | 63.48 |
| 29.0 | -193 | 29.70 | 19.17 |
| 14.72 | 14.72 | 14.72 | 14.72 |

TABLE III
STRUCTURE C (0.125mm STEP SIZE)

| Actual values | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| ELECTRIC CONDUCTIVITY (S/m) | | | |
| 0.0369 | 0.0369 | 0.0369 | 0.0368 |
| 0.7192 | 0.7192 | 0.7192 | 0.7116 |
| $1.160 \cdot 10^6$ | $0.135 \cdot 10^6$ | 1234. | -0.1164 |
| 0.0674 | 0.0674 | 0.0674 | 0.0 |
| RELATIVE ELECTRIC PERMITTIVITY | | | |
| 5.922 | 5.922 | 5.922 | 5.898 |
| 63.47 | 63.47 | 63.46 | 60.80 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 14.72 | 14.72 | 14.72 | 0.0 |

TABLE IV
STRUCTURE C (0.03125mm STEP SIZE)

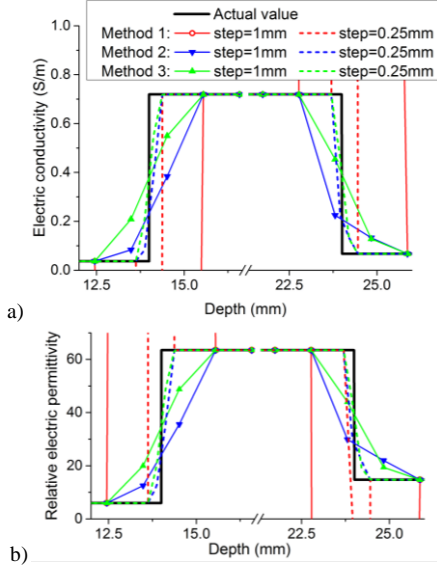| Actual values | Method 1 | Method 2 | Method 3 |
|---|---|---|---|
| ELECTRIC CONDUCTIVITY (S/m) | | | |
| 0.0369 | 0.0369 | 0.0369 | 0.0369 |
| 0.7192 | 0.7192 | 0.7192 | 0.7192 |
| $1.160 \cdot 10^6$ | $1.155 \cdot 10^6$ | $0.947 \cdot 10^6$ | $1.092 \cdot 10^6$ |
| 0.0674 | 0.0674 | 0.0674 | 0.0 |
| RELATIVE ELECTRIC PERMITTIVITY | | | |
| 5.922 | 5.922 | 5.922 | 5.919 |
| 63.47 | 63.47 | 63.46 | 63.00 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 14.72 | 14.72 | 14.72 | 0.0 |



Fig. 1. Reconstruction of electric conductivity (a) and relative permittivity (b).

$$\frac{d}{dz}\left(\frac{dH}{dz}u\right) + \omega^2 \mu_0 H = 0 , \tag{3}$$

that suggests a finite difference centred scheme, using the same grid over which the magnetic field has been evaluated.

## IV. RESULT DISCUSSION

The first considered structure (A) includes three materials with properties similar to the ones of some human tissues at the frequency of 128 MHz. A fourth medium, added in place of part of the third layer, can have a very low (B) or a very high electric (C) conductivity to simulate the presence of ceramic or metal objects, respectively.

The analysis is first developed with a 1 mm step in structure A, where all methods seem to well run when considering the averaged values of the electric parameters (see Table I). However, the limits of the methods are put in evidence by the spatial distributions of parameters (Fig. 1) near the interfaces between tissues: Method 1 shows very large oscillations, while Methods 2 and 3 can only approximate the stepped behavior. The results for Methods 2 and 3 (0.25 mm) improve with a finer mesh, but the oscillations of Method 1 are not damped.

A very low conductive material (structure B) makes the reconstruction more difficult. With a 1 mm step the averaged values (Table II) show that no method is able to reconstruct the actual conductivity in the low conductive material and that only Method 2 reconstructs exactly the relative permittivity of all the materials. A good agreement in conductivity is found reducing the step to 0.25 mm.

Due to the strong skin effect, structure C requires finer meshes. Strong instabilities are always found in Method 3, whose results are untrustworthy, anyway Methods 1 and 2 underestimate the conductivity with a 0.125 mm step (Table III). A good reconstruction is reached only with a step finer than the metal penetration depth (Table IV).

## V. REFERENCES

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